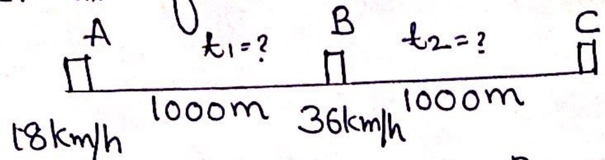
1. A train is uniformly accelerated and passes successive kilometre stones with velocities of 18 km/h and 36 km/h respectively. Calculate the velocity when it passes the third kilometre stone. Also find the time taken to each of these two intervals of 1km.



Considering motion from A to B

$u = 18\,km/h = 5\,m/s$

$V = 10\,m/s$

$S = 1000\,m$

$V^2 = u^2 + 2aS$

$$a = \frac{V^2 - u^2}{2S} = \frac{100 - 25}{2\times1000} = \frac{75}{2\times1000} = 0.0375\,m/s^2$$

$V = u + at$

$$t_1 = \frac{V-u}{a} = \frac{10-5}{0.0375} = \underline{133.33\,s}$$

$$V_c^2 = u^2 + 2aS$$

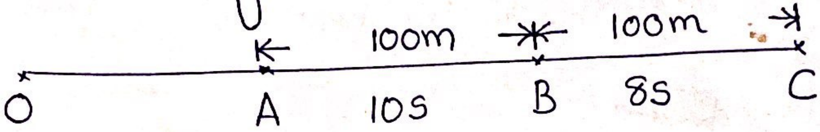
$$= 10^2 + 2\times0.0375\times1000$$

$$= \sqrt{175} = 13.22\,m/s.$$

$V = u + at_2$, $\forall\ t_2 = \frac{V-u}{a} = \frac{13.22 - 10}{0.0375} = \underline{86.1\,s}$

2. 3 marks A, B & C at a distance of 100m each are made along a straight road. A car starting from rest and moving with uniform acceleration passes the mark A and takes 10 s to reach B and further 8 s to reach C.

i) Calculate the magnitude of acceleration of the car
ii) Velocity of car at A
iii) Velocity of car at B
iv) Distance of A from the starting point.



A to B

Initial velocity $= V_A$

Final velocity $= V_B$, $S = 100m$, acceleration $= a$

$t = 10s$.

$S = ut + \frac{1}{2}at^2$

$100 = V_A \times 10 + \frac{1}{2} \times a \times 10^2$

$200 = 20 V_A + 100a$ — ①

A to C

Initial velocity $= V_A$, $S = 200m$

$t = (10+8) = 18s$

$S = ut + \frac{1}{2}at^2$

$200 = V_A \times 18 + \frac{1}{2} \times a \times 18^2$

$400 = V_A \times 36 + 324a$ — ②

$V_A = 8.611\ m/s$, $a = 0.277\ m/s^2$

$V_B^2 - V_A^2 = 2aS$

$V_B^2 - (8.611)^2 = 2 \times 0.0278 \times 100$

$V_B = 11.39 m/s$.

iv)

$v^2 - u^2 = 2as$

$u = 0$, $V = 8.61 m/s$, $a = 0.0278\ m/s^2$

$8.61^2 = 2 \times 0.0278 \times S$

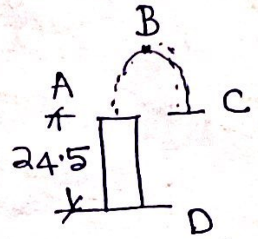
$S = 133.33m$

3. A stone is thrown vertically upwards with a velocity of 19.6 m/s from the top of the tower 24.5 m height. Calculate

i) The time required for the stone to reach the ground

ii) Velocity of the stone in its downward travel at the point in the same level as the point of projection.

iii) Maximum height to which the stone will rise in its flight



Given $u = 19.6 m/s$

$V = 0$

$a = -g$ (upward motion)

Time to reach maximum height from the point of projection

$V = u - gt$

$t = \frac{V-u}{-g} = \frac{0-19.6}{-9.8} = 1.997s \approx 2s$

ii) The distance to maximum height from top of tower

$S = ut - \frac{1}{2}gt^2$

$= 19.6 \times 2 - \frac{1}{2} \times 9.81 \times 4 \quad = 19.58m.$

The maximum height from ground $= 19.58 + 24.5 = 44.08$

Consider B to C

$u = 0$, $t = 2s$. $V = ?$

$V = u + gt$

$= 0 + 9.81 \times 2 = \underline{\underline{19.62}}$

So velocity at C = velocity at the point of projection

<u>C to D</u>

$S = 24.5m$, $u = 19.62 m/s$

$S = ut_3 + \frac{1}{2}gt_3^2$

$24.5 = 19.62t + \frac{1}{2} \times 9.81 t^2$

$t = 1$ sec.

Total time $t = 1+2+2 = \underline{5 \text{ sec}}$

4. A stone is dropped into a well is heard to strike the water after 4 sec. Find the depth of water in the well if the velocity of sound is 350 m/s.

$t = t_1 + t_2 = 4$ sec.

For the stone

$S = d$

$u = 0, \quad d = ut + \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8 t_1^2 \quad — ①$

For the sound

$d = vt_2$

$= 350 t_2$

$t_1 + t_2 = 4 \quad \therefore t_2 = (4 - t_1)$

$d = 350(4 - t_1) = 1400 - 350 t_1 \quad — ②$

$4.9 t_1^2 = 1400 - 350 t_1$

$4.9 t_1^2 + 350 t_1 - 1400 = 0$

$t_1 = 3.798$

$d = 70.38$ m

5. A cage descends a mineshaft with an acceleration of 0.6 m/s². After the cage has travelled 30 m, a stone is dropped from the top of the shaft. Determine i) the time taken by the stone to hit the cage & ii) the distance travelled by the cage before impact.

Let t time for stone in motion, S distance from top to point where impact takes place.

$S = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.81t^2$ —①

Let t_1 time taken to travel 30m. by the cage

$a = 0.6\, m/s^2$, $u = 0$.

$30 = \frac{1}{2} \times a t_1^2 = \frac{1}{2} \times 0.6 \times t_1^2$

$t_1 = 10$ sec.

When stone strikes, cage has travelled $(t+10)$ sec.

$\therefore S = 0 + \frac{1}{2} \times 0.6 \times (t+10)^2$ — ②

Substituting ① in ②

$\frac{1}{2} \times 9.81 \times t^2 = \frac{1}{2} \times 0.6\,(t+10)^2$

$4.9\,t^2 = 0.3\,(t^2 + 100 + 20t)$

$4.6t^2 - 6t - 30 = 0$

$t = 3.288$ sec.

$S = \frac{1}{2}gt^2$

$= \frac{1}{2} \times (9.81) \times 3.288^2 = \underline{53.028m}$

6. A body is moving with uniform acceleration and cover 15m in the 5th second and 25m in the 10th sec. Determine the initial velocity of the body and acceleration of the body.

$S_5 = 15 = u + \frac{a}{2}(2 \times 5 - 1)$

$15 = u + \frac{9a}{2}$ —①

$S_{10} = 25 = u + \frac{a}{2}(2 \times 10 - 1)$

$25 = u + \frac{19a}{2}$ —②

② − ① $\Rightarrow 10 = 5a$

$a = 2\, m/s^2$

$a = 2$ in (1) $\Rightarrow 15 = u + 9$

$u = \underline{6\, m/s}$

Kinetics

1. A body of mass 500kg is moving with a velocity of 25m/s. A force of 200N acts on it for 2 minutes. Find the velocity of the body when
a) Force acts in the direction of motion
b) Force acts in the opposite direction of motion.

$m = 500kg$
$u = 25m/s.$
$F = 200N$
$t = 2 \text{ minutes} = 2 \times 60 \text{ sec}.$

a) $a = \frac{F}{m} = \frac{200}{500} = 0.4 \, m/s^2$

$\frac{v-u}{t} = a$

$v = u + at$
$= 25 + 0.4 \times 2 \times 60$
$= 73 m/s.$

b) $a = \frac{-F}{m} = \frac{-200}{500} = -0.4 \, m/s^2$

$v = u - at$
$= 25 - 0.4 \times 2 \times 60 = -23 m/s$

-ve sign shows the direction of motion of the body get reversed.

2. A force 100N acts on a body having a mass of 4kg for 10 seconds. If the initial velocity of body is 5 m/s determine the acceleration produced in the direction of force and distance moved by the body in 10sec.

$F = 100 N$
$m = 4kg$

$a = \frac{F}{m} = \frac{100}{4} = 25 m/s^2$

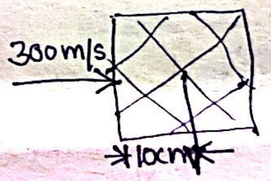
$t = 10 sec.$

$u = 5 m/s.$

$S = ut + \frac{1}{2}at^2$

$= 5 \times 10 + \frac{1}{2} \times 25 \times 10^2 = \underline{1300 m}$

3. A bullet of mass 81g is moving with a velocity of 300 m/s is fired into a log of wood and it penetrates into a depth of 10cm. If the bullet moving with the same velocity is fired into a similar piece of wood 5cm thick, with what velocity would it emerge? Find also force of resistance offered by the wood, assuming it to be uniform.



Case I

$m = 81g = 0.081 kg$

$u = 300 m/s$

$V = 0$

$S = 10 cm = 0.1 m$

$V^2 = u^2 + 2aS$

$a = \frac{-u^2}{2S} = \frac{-300^2}{2 \times 0.1} = -45 \times 10^4 m/s^2$

Case II Both materials are same, so a is same for both cases.

$a = -4.5 \times 10^5 m/s^2$

$u = 300 m/s$

$S = 5cm = 0.05 cm$

$V^2 = u^2 + 2aS = 300^2 + 2 \times -4.5 \times 10^5 \times 0.05$

$= 212.1320 m/s$

F = all the forces acting along the direction of motion.

$F = -R = m \times -a$

$R = 0.081 \times 45 \times 10^4 = \underline{3.645 \times 10^4 N}$

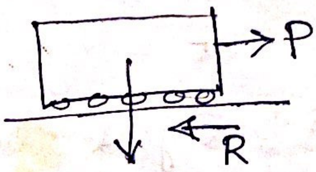
4. A multiple unit electric train has 800 tons mass. The resistance to the motion is 1% of train's mass. The electric motor can provide a pulling force of 200kN. How long does it take to accelerate the train to a speed of 900 km/h on a level track.

1ton = 1000kg

Resistance, $R = 0.10 \times 800$ ton

$= 0.10 \times 800 \times 10^3$

$= 8000$ kg



Resistive force $= 8000 \times 9.81$

$= 78480N$

By Newton's second law

$F = ma$

$F = P - R$

$= 200 \times 10^3 - 78.480 \times 10^3 = 121.52 \times 10^3$ N

$ma = 121520$

$a = \frac{121520}{8 \times 10^5} = 15190 \times 10^{-5} = 0.151 m/s^2$

$v = u + at$

$u = 0$

$v = 900$ km/h $= 900 \times \frac{5}{18} = 250$

$250 = v = 0 + at = 0 + a \times t$

$t = \frac{250}{0.15} = 1666.6$ sec. $= 27.7$ min.

5. A car of mass 2.5 tons moves on a level road under the action of 1kN propelling force. Find the time taken by the car to increase the velocity from 36 km/h to 54 km/h.

$m = 2.5t = 2.5 \times 10^3$ kg

$F = 1KN$

$F = ma$ by Newton's II Law

$1 = ma$

$a = \frac{1}{m} = \frac{1 \times 10^3}{2.5 \times 10^3} = 0.4 \ m/s^2$

$u = 36 \times \frac{5}{18} = 10 \ m/s.$

$V = 54 \times \frac{5}{18} = 15 \ m/s.$

$t = \frac{V-u}{a} = \frac{15-10}{0.4} = \underline{\underline{12.5 \ sec.}}$

6. A body of mass 10kg is moving over a smooth surface whose equation of motion is given by the relation $S = 5t + 2t^2$ where 's' in 'm' and 't' in seconds. Find the magnitude of force responsible for the motion.

$S = 5t + 2t^2$

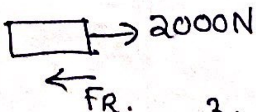
$V = \frac{ds}{dt} = 5 + 4t$

$a = \frac{dv}{dt} = 4 \ m/s^2$

∴ By Newton's second law $F = ma$

$= 10 \times 4 = \underline{40N}$

7. The tractive force exerted by a railway cart weighing 50kN is 2000N. If the frictional resistance is 5N/kN of the railway car's weight, determine the acceleration when the railway cart is moving on a level track.



$F_R = 5N/kN$ ∴ $5 \times 50 = 250N$

$W = 50kN$

$m = \frac{W}{g} = \frac{50 \times 10^3}{9.81} = 5.096 \times 10^3 \ kg$

$F = P - F_R$

$a = \frac{2000 - 250}{5.096 \times 10^3} = \underline{\underline{343.4 \times 10^{-3} \ m/s^2}}$

8. A train of weight 1960kN starts from rest and attains a speed of 120 km/hr. in 5 minutes. If the frictional resistance of the track is 10N/kN of train's weight, find the average

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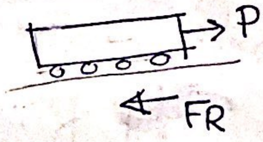
pull required along level track.



$W = 1960\,kN$

$u = 0$

$V = 120 \times \frac{5}{18} = 33.33\,m/s$

$t = 5\,min = 5 \times 60 = 300\,sec.$

$a = \frac{V-u}{t} = \frac{33.33-0}{300} = 0.11\,m/s^2$

$F_R = 10N/kN \therefore 10 \times 1960 = 19600N$

$F = ma$ by Newton's second law

$P - F_R = ma$

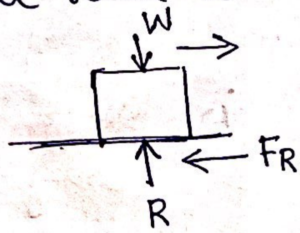
$m = \frac{1960 \times 10^3}{9.81} = 199.79 \times 10^3\,kg$

$P = ma + F_R$

$= 199.79 \times 10^3 \times 0.11 + 19600$

$= 41.5769 \times 10^3\,N$

9. A car moving on a straight level road skidded for a total distance of 60m after the brakes are applied. Determine the speed of the car just before the brakes are applied if the μ between car's tyre and road is 0.4. Take $g = 9.8\,m/s^2$.



$u = ?$

$V = 0$

$\mu = 0.4$

Net force $= -F_R$

$= -\mu R$

$= 0.4 \times W$

$ma = 0.4 \times W$

$\left(\frac{W}{g}\right) a = -0.4 \times W$

$a = -0.4 \times 9.8$

$= -3.92\,m/s^2$

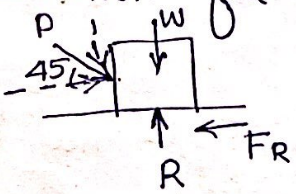
$V^2 = u^2 + 2as$

$u^2 = -2as$

$u = \sqrt{-2 \times -3.92 \times 60} = \underline{21.68\ m/s}$

T-1 CSB. 3018

10. A body of mass 50kg rest on a rough horizontal surface. [μ = 0.4], it is acted upon by a force applied at an angle of 45° with horizontal. Determine the magnitude of force if it causes the body to move with an acceleration of 2 m/s².



Net force causing motion

$= P\cos 45 - \mu R$

$= P\cos 45 - \mu[50 \times 9.81 + P\sin 45]$

By Newton's II Law

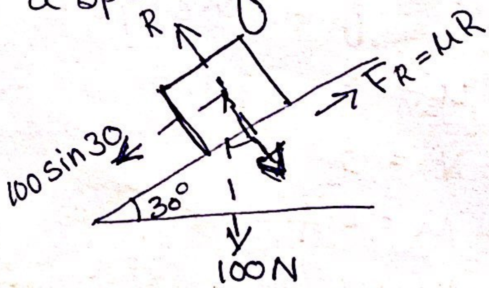
$50 \times a = P\cos 45 - \mu(50 \times 9.81 + P\sin 45)$

$P(\cos 45 - 0.4\sin 45) = 50 \times 2 + 0.4 \times 50 \times 9.81$

$P = \dfrac{100 + 0.4 \times 50 \times 9.81}{\cos 45 - 0.4 \times \sin 45}$

$= \underline{698.15N}$

11. A body of weight 100N is initially stationary on a 30° inclined plane. What distance along the inclined plane must the body slide before it reaches a speed of 2.5m/s. μ b/w body and plane is 0.1



Net force causing motion

$= 100\sin 30 - \mu \times 100\cos 30$

$= 100\sin 30 - 0.1 \times 100\cos 30$

$= 100\sin 30 - 0.1 \times 100\cos 30$

$= 41.34 = ma$

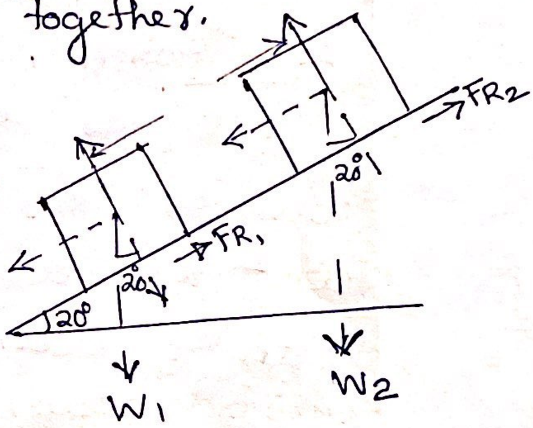
$m = \dfrac{W}{9.8} = 10.52$

$a = \dfrac{41.34}{10.52} = 4.06\ m/s^2$

$u = 0,\ v = 2.5\ m/s$

$S = \dfrac{v^2 - u^2}{2a} = \dfrac{2.5^2}{2 \times 4.06} = 0.77m$

12. Two bodies 10m apart are held stationary on an inclined plane having an inclination of 20°. μ b/w the lower body and the plane is 0.08 and that b/w the upper body and plane is 0.05. If both the bodies are set in motion at same instant, calculate the distance through which each body travel before they meet together.



Let the required distance for

lower one $d_1 = x$

For $d_2 = 10 + x$

Net force acting on lower body

$= W_1 \sin 20 - 0.08 \times W_1 \cos 20$

$= W_1 (\sin 20 - 0.08 \times \cos 20) = m_1 a$

$= \frac{W_1}{g} a_1$

$a_1 = 9.8 [\sin 20 - 0.08 \times \cos 20] = 2.615 \, m/s^2$

$u = 0$

$S = \frac{1}{2} a t^2 \quad \therefore t = \sqrt{\frac{2S}{a}} = \sqrt{\frac{2x}{2.62}}$

$x = \frac{1}{2} a t^2 = 1.31 t^2 \quad -①$

For upper body

$F_2 = W_2 \sin 20 - 0.05 \times W_2 \cos 20$

$\frac{W_2}{g} a_2 = W_2 (\sin 20 - 0.05 \cos 20)$

$a_2 = g [\sin 20 - 0.05 \cos 20]$

$= 2.89 \, m/s^2$

$10 + x = \frac{1}{2} \times 2.89 \times t^2$

$= 1.445 t^2$

$x = 1.445 t^2 - 10 \quad -②$

$① - ② \Rightarrow t^2 (1.31 - 1.445) = 10$

$$t = \sqrt{\frac{10}{1.31 - 1.445}} = 8.60 \text{ sec.}$$

$$\therefore x = 1.31 \times 8.6^2$$

$$= 97.03 m$$

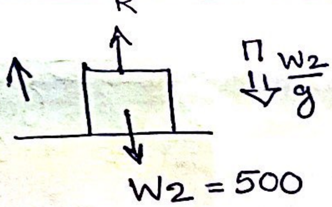
For upper body $s = 10 + x$

$$= 10 + 97.03 = \underline{107.03 m}$$

Lift motion

1. A lift has an acceleration of $1.225 m/s^2$. What force will a man weighing 500N exert on the floor of the lift. If i) the lift is moving up what force would be exerted with an acceleration of $1.225 m/s^2$ ii) downwards iii) What upward acceleration would cause this weight to exert a force of 600N on the floor.

Case (i)



Net Force if the lift is moving upwards with a of $1.225 m/s^2$ $= R - W_2$

By Newton's second law

$$F = ma$$

$$R - W_2 = m.a = \frac{W_2}{g} . a$$

$$R = W_2\left(1 + \frac{a}{g}\right) = 500\left(1 + \frac{1.225}{9.81}\right) = \underline{562.436 N}$$

Case (ii) ↓

$$F = ma$$

$$W_2 - R = \frac{W_2}{g} . a$$

$$R = W_2\left(1 - \frac{a}{g}\right) = 500\left(1 - \frac{1.225}{9.81}\right) = 437.56 N$$

iii) a = ? ↑

when R = 600N

$$R - W_2 = \frac{W_2}{g} a \quad (\because F = ma)$$

$$a = (R - W_2) \times \frac{g}{W_2} = (600 - 500) \times \frac{9.81}{500} = \underline{1.962 m/s^2}$$